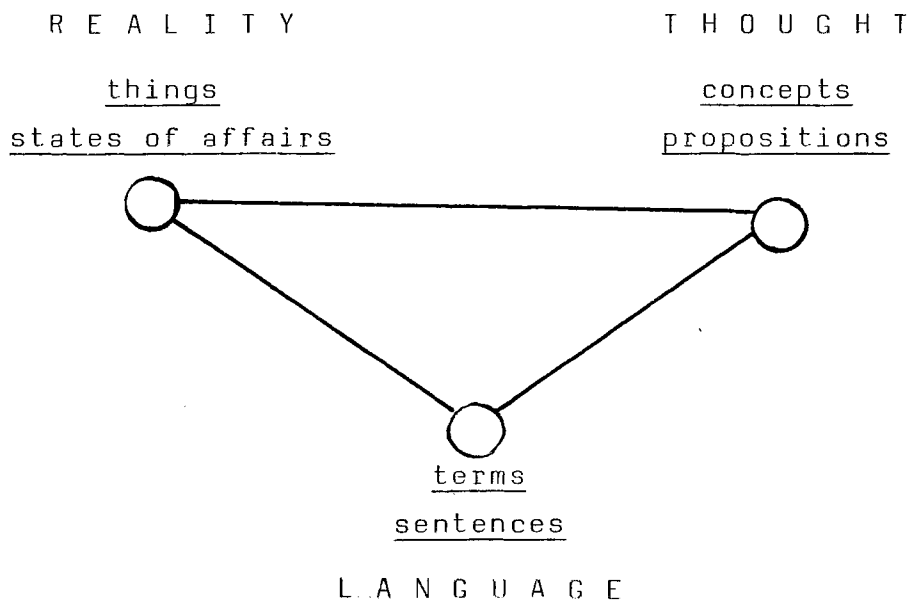


4.4 Contradictory reality and mathematics: A contradiction?

At this point let me try to relate back to the first section of the first chapter on christian and buddhist epistemology. Theories, certainly including theories built around human needs, are formulated in languages. They are expressions of thought about some aspect of reality. And that brings up the whole question of relationship between reality, thought and language, a rather basic problem in philosophy, and certainly a problem about which no philosopher has been able to come up with anything like a final answer (not by that implying that there are final answers to anything at all).

The present effort to explore this trilateral relationship will take as its point of departure that classical effort to keep the three apart, Ogden's triangle¹ (drawn here perhaps in a somewhat unusual way):

FIGURE 1. Ogden's triangle, separating things-concepts-terms



A term in a language has a denotation, the thing it denotes; and a connotation, the concept it represents. At a more complex level a sentence denotes a state of affairs and connotes a proposition. At a still more complex level we get into texts and contexts. A positive, indeed very useful, aspect of Ogden's triangle can now be formulated: it is symmetric, equilateral, can be turned around with any corner up top. There is no built in assumption that any corner is primary, the other two being derivative. That also reflects the agnosticism of the present author on that issue: I do not see sufficient reason to assume that language unambiguously shapes thinking or constructs reality just as little as I would assume that language is the perfect verbalization of man's encounter with reality, first reflected in thoughts. Nor would I assume any basis for thought as a clearing house between reality and language. Rather, I would see mutual influence, impact, even steering, but always filled with ambiguity and latitude, in all relations in the triangle.

Let us now try to cut into this triangle with one basic theme of analysis: contradiction vs. consistency, looking at this theme from the three corners of the triangle.

Reality and contradiction vs. consistency

I am not in a position to say what reality is; all I think I can say something about is how reality is constructed. However, I do not relate those constructions to thought and language as such, but rather to deeper lying assumptions in a civilization,

what is customarily referred to as "religion". Constructions of reality then become explications of those assumptions.

Let me give two personally experienced examples of ways of imputing what might even be called basic contradictions to reality. The first example picks up the buddhist application of the principle of impermanence, anicca, to human beings in the doctrine of anatta, the assumption that human beings do not possess a permanent Self. On the other hand, buddhism also has a doctrine of rebirth in the sansara cycle, whereby the vital forces re-cohere in another human being.²

Learning this in a buddhist community in Southeast Asia I went to the bhikkhu, the monk and complained: this is a contradiction. His answer, to my surprise, was neither to deny that there was a contradiction, nor to resolve the contradiction through what I expected, reinterpretation of such crucial concepts as "Self", "rebirth"; or the anatta doctrine and the rebirth doctrine. What he said was this: "yes, there is a contradiction there, and with that contradiction you can live, you can work on the contradiction and the contradiction will work on you".

Leaving aside the implication of that statement for buddhist philosophy let me point to the general implication: an approach to contradiction excluding denial and resolution on the one hand, and passive acceptance of the contradiction on the other. I shall return to this point, only noting what here on the printed

page might look very naive: the sentence struck me (with some eight years of university studies in mathematics and considerably more than that of initiation in western civilization) as a lightening bolt. Let me only add that from the bhikkhu the sentence came just like that, an everyday statement, as a matter-of-fact--and it was certainly not pronounced by a person with university training in philosophy or anything similar.

My second example comes from the same part of the world but is political rather than religious. What I now quote summarizes a number of statements I have heard from Chinese discussing political economy, and I claim that my summary is fairly typical of Chinese thought. The statement goes approximately as follows:

"Capitalism is very good because it generates economic growth, building on the initiative of those who are capable of entrepreneurial activity, including taking risks. But capitalism is very bad because it generates very rich people and very poor people and the rich people tend to become too powerful and in addition corrupt, and the poor people also become materialistic and consumption oriented and often unemployed, destitute and prostitute, given to criminal acts of various kinds. Consequently, we need socialism, which is very good because it is based on planning so as to satisfy the basic needs of those in need while at the same time putting a lid on the very rich people, thereby making for a more egalitarian society that can be devoted to less materialistic goals. However, socialism is very bad because it leads to too much concentration of power in the hands of the state and consequently becomes repressive and in addition static, nondynamic. For that reason we need capitalism which is very good because-----"

In a simplified form the structure of the argument can be given as a fourfold table:

In any entity there is yin and yang, opposed forces or principles. However, in yin there is yin and yang and in yang there is yin and yang so that the yin in yin is dominant and the yang in yin is recessive (otherwise we would be dealing with yang) and yin in yang is recessive whereas the yang in yang is dominant (otherwise we would be dealing with yin). And thus it continues, as the saying goes with "elephants all the way down". Translated to the case of Figure 2 one might say that in an entity like a society capitalism and socialism should not be seen as dominating the ground alone but as being complementary principles or opposed forces if one prefers more adversarial language. However, inside capitalism there are good and bad forces and inside socialism there are good and bad forces. By adequate translation some of this may perhaps be brought on a form more reminiscent of standard interpretations of yin and yang as female/male, dark/light. But this seems unimportant, epistemologically uninteresting.

The basic point is the following: a statement about a society can never be in terms of giving that society one attribute alone unless that attribute should be the term "contradictory". Doing so, however, does not add to our knowledge, it is a truism within this universe of discourse. What is not a truism is to spell out the contradictory forces or principles, for instance in the way it is done in Figure 2. But it should be noted that Figure 2 only brings us to Level 2 of exploration. To this a Level 3 could be added exploring contradictions within all four cells' of Figure 2. Example: inequality in capitalism is

mainly bad (dominant characteristics) but is also good (recessive characteristics): it stimulates work among those who want to catch up. This, in turn, is mainly good but it is also bad: it makes for too much ambitiousness, competitiveness, individualism with lack of solidarity, and so on (the reader will note that we are right now at Level 4 of analysis).

Does this not mean that the generalized yin/yang scheme reduces to a pro et contra scheme? I think there is a difference: the latter refers to language, the yin/yang scheme to reality. These forces are real in the sense of operating independent of human reflecting upon them. But then there is another difference, and this may perhaps be indicated with a simpler example.

Classical Chinese thought seems to be hesitant in attributing to a thief the attribute guilty. In guilt there is certainly guilt but also an element of nonguilt, and every person is not only guilty but also nonguilty. A westerner might reply: that means the person is guilty in some respects, and nonguilty in others. He might look at the person as a unit moving in a continuum of time and space, analyzable in terms of a number n of variables, and as he moves in time and space certificates might be issued, mapping each time-space point for each variable on a simple set of two elements, G for guilty and N for nonguilty. The process of doing this might be referred to as analysis, and the yin/yang description of the person as guilty and nonguilty will simply be seen as sloppy

description, crude, to be dissolved into its components when exposed to the prism of adequate analysis.

I think a daoist answer to this might be that the basic point has been lost. There is no denial that upon scrutiny, with a finer grid of analysis, the ratio of guilty to non-guilty may change; indeed, a basic assumption is that it would change along the time axis, it is when deviation from a point of balance is excessive (note that this would mean that the totally nonguilty person is not considered harmonious or ideal, an interesting point to contemplate. Even virtue, in excess, becomes a vice--) that problems arise.

But why is it so important that reality should not be contradictory to this homo occidentalis brought into our text? Probably it has something to do with the image of what the Creator created as consistent, contradiction-free. Of course, aspects of reality may look contradictory to us but that is only because we do not understand them. From the highest vantage point there is a consistent plan underlying the universe but fully understandable only to God himself. That understanding can be approximated, though, but only in humility, only by praying to God for a higher level of consciousness, and even so not necessarily granted.

Thought and contradiction vs. consistency

Cutting across the difference in epistemology as they have been constructed in ch. 1.1 above is the principle of adequatio:

there has to be a basic isomorphism between reality and thought. If reality is contradiction free thought should also be contradiction free, whence the classical Laws of thought:

1. No contradiction: $\sim (p \ \& \ \sim p)$
(consistency)
2. Excluded middle: $\sim p \vee p$
(tertium non datur)
3. Principle of identity: $(p) \ p = p$

Applying the Laws of thought to the Laws of thought themselves the tertium non datur principle coincides with to the consistency principle and the principle of identity is not significant for our purpose. Hence, I am left with the theme of this exploration, contradiction vs. consistency.

The principle is important because the negation of the principle makes unambiguous logical deductions impossible. In classical analysis the relation of implication, "proposition p implies proposition q" ($p \longrightarrow q$) is explicated as follows:

p	q	$p \longrightarrow q$
true	true	true
true	false	false
false	true	true
false	false	true

Proposition p can be true or false, so can proposition q, giving us four possibilities. Their relationship, p implies q, holds in three of them but not in the fourth: it does not hold if p is true and q is false. If, now, p is true, and $p \longrightarrow q$, then it

follows that q true (modus ponens). If q is false then it follows that p is also false (modus tollens). If p is false nothing definite follows about q and if q is true nothing definite follows about p.

Terribly elementary and terribly important--in western logic. The moment it is assumed that p or q or both can be both true or false no basis any longer exists for implication, and if this relationship is seen as basic in theory formation then theory formation is also out because deduction is out. Admitting contradictions somewhere in the system opens for contradiction everywhere. But in that case a theory will no longer exclude anything, which means that no statement can be validated or invalidated on logical grounds. And this, in turn, means that theories cannot be used to capture reality, holding it so to speak within the confines of the theory. In other words, reality will no longer have any permanence of any kind, static or dynamic, as reflected in the theory. Reality is neither accounted for, nor accountable to, the theory.

A sense of consistency/inconsistency, from this angle, now becomes a condition not only for following the Laws of thought but of being capable of doing so. If these laws are seen as in need of no justification, as apodictic categories, then the only possible conclusion would be that he who does not reason according to the Laws of thought is incapable of thinking, in other words deficient. Consistency deficits becomes a mental health deficit, and from there the road to psychiatrization may be opened. To use the two examples above: "I have no soul and I

am going to be reborn" makes sense in a buddhist context but not in a christian context; "I believe in capitalism and socialism" makes sense in a Chinese context but considerably less so in a U.S. context. With a little bit of patience, however, the ground may be cleared for a fruitful dialogue in either case.

Language and contradiction vs. consistency

We may now make use of the distinction between hard and soft languages, following Nalimov. I shall quote him at some length:⁴

A hard language is here seen as a language not permitting any ambiguity, in other words a consistent or contradiction-free language. I would then see logic in general and mathematics in particular as a language the speakers of which have entered a community, a corpus mysticum by signing a pact: I may choose the (primitive) terms I want, define the (primary) sentences (axioms) I want, and then use the Laws of thought; but always in such a way that I shall never deduce a sentence (theorem) and its negation. The axioms themselves have to be consistent; in addition there should be no redundancy in the set of axioms (they are all necessary) and the set of axioms should be complete (they are sufficient as a basis for deciding, for every correctly formulated sentence in that "dialect" of the mathemat-

ical language, whether the sentence is true or false).

I understand the famous theorem by Gödel (1930)⁵ to say the following: this is not in general possible. Under certain conditions theorems can be formulated that are undecidable; it cannot be decided whether they are true or false. I do not see that this is tantamount to saying that Gödel proved that mathematics cannot be contradiction free; what he proved was that we cannot prove that certain parts of mathematics are contradiction free, nor that they are contradictory.

Contradictions enter mathematics, however, through paradoxes of the classical Epimenides type, the famous Cretan reputed to have said "All Cretans are liars", and the problem arises the moment it is ascertained that Epimenides himself is a Cretan. Bertrand Russell's theory of types, excluding from set theory all sets that have themselves as members (thereby confusing the level of element and the level, or type, of set) is seen as a solution to the problem of paradoxes by ruling certain sentences out of the language of mathematics as incorrectly spoken, as certainly also happens in natural languages. In doing so Russell functions, with the consent of his colleagues, much like a language academy, introducing or reenforcing traffic rules in that language community.

I see no reason why speakers of a language should not be permitted to do so, only take note of the obsessive concern with contradictions. That this has been seen as a limitation

is also clear from the effort to develop the language of polyvalent logic, with more than two truth values. However, if the Laws of thought used in deriving theorems in polyvalent logic still are based on bivalent logic then it is hard to see that any exception has been made to the rules states above: in mathematics everything is permitted except contradictions.

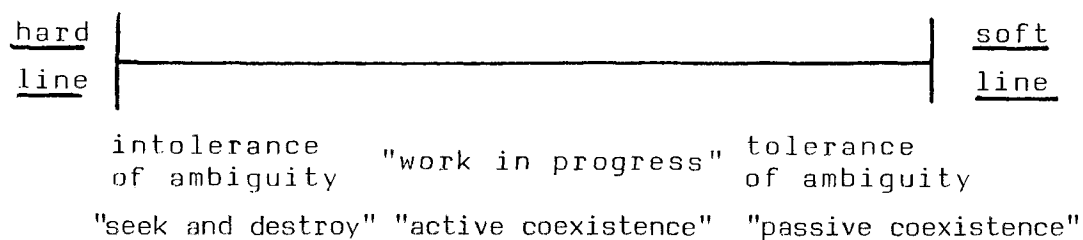
What about natural languages? Indo-european languages may be said to be predicative, attributing a predicate to the substantive, whereas Chinese and Japanese may be said to be relational, postulating relationships where the two terms (or four terms in the classical Chinese quartet form of linguistic presentation) enter symmetrically. One is not an attribute of the other.⁶ However, even if this opens for more holistic, dialectical thinking, placing Chinese and Japanese at higher levels of abstraction (two place, even four place as opposed to one place logic) this difference does not provide an escape from the consistency constraint. That escape, at least as far as Japanese is concerned, is provided in another way: through a certain vagueness.⁷ Japanese language, when translated into european languages, tends to be punctuated with a high number of such qualifiers as "perhaps," "maybe." In other words, certificates of truth or falsehood in the western dichotomous sense, one or the other, are not readily issued. Does "perhaps" stand for a truth value between true and false, or for true and false at the same time, or for both of these, or for neither? I do not think there is any clear answer to this question either; maybe all three?

But there is a consequence of this observation: a possible hesitation in the Japanese language community when it comes to theory formation. This does not mean that Japanese cannot become as good or better mathematicians than anybody else; in fact, they tend to excel also in this field.⁷ What it does mean might be a hesitation in conceiving of mathematics as an adequate language in which contradictory or at least contingent ("perhapsy", "maybeish") reality can be reflected. Everything becomes too crude, too clear-cut. And the same would also apply to the use of natural language: Japanese would tend to be very concrete, sticking to the grounds so to speak with detailed descriptions and collection of data rather than engaging in speculations and theory formation. As a matter of fact, I am not convinced that behind the Japanese economic miracle one will find many mathematical models, perhaps not even computers.

Three approaches to contradictions

Let us now return to the bhikkhu mentioned above and try to see his statement in a broader perspective:

FIGURE 4. Approaches to contradictions



Three approaches have been indicated, with the obvious conclusion that the author stands for the one in the middle, as one more implication of the in media res principle ("Vernunft in der Mitte"). Social democracy as opposed to capitalism/socialism.

At one extreme is what is seen as a characteristic of occidental epistemology: intolerance of ambiguity. Wherever contradictions are found the appropriate response is "seek and destroy". In mathematics this takes the form of traffic rules. In natural languages it takes the form of rules for the correct use of language, certain things should not be said (like "I am a liberal, and I am a marxist"), among other reasons because they may be indicative of deeper confusions at the level of thought and lead to insanity certificates being issued. In relations to reality it takes the form of purification of reality, sorting good from evil which then opens for particular approaches to politics of all kinds, and so on. The "seek and destroy" formula is not chosen at random: it stands for purifying territory, for instance in Vietnam, of enemies, for instance "communists".

At the other end of the spectrum, then, is perfect tolerance of ambiguity. A contradiction does not serve as any admonition to any kind of action, neither on reality, nor in the organization of speech, nor in the disciplining of thought. The person coexists passively with the contradiction, nothing happens; there is quiescence.

The position in the middle, then, is characterized by "active coexistence", to stick to this particular political language. There are tensions, they are recognized. But two approaches are studiously avoided: the two extremes of the continuum. A contradiction serves as the signal for activity in the direction of overcoming contradiction. However, a far greater danger than coexistence with the contradiction is brushing it under the carpet with a glib "solution". The Russell theory of types is probably of this kind; no doubt other approaches have been found and will be found leading to much deeper insights. Working for societies that are purely capitalist or purely socialist is an important occidental pastime, and the perfect expression of the hard line approach to contradictions; but equally perfect examples of efforts to overcome contradictions too easily, too quickly. On the other hand, however, the fatalistic quiescence of simply tolerating any ambiguity there is around is also seen as unsatisfactory.

A supreme example of one theoretician who opted for this third approach is, of course, the Danish physicist Niels Bohr. In his complementarity principle two languages are permitted at the same time, one emphasizing the continuity of light phenomena another the discontinuity--also expressed as the wave and particle approaches respectively.⁸ If reality exhibits contradictory properties then why not use contradictory or mutually exclusive languages to reflect this state of affairs? Characteristically Niels Bohr adopted the daoist yin/yang symbol as his personal coat of arms, and also indicated in one of his

writings that this approach might be particularly valid for "living organisms and the characteristics of people having consciousness as well as human cultures themselves"? However: this is hardly the last word to be said in these matters, the principle of complementarity might perhaps rather be seen as an indication of "work in progress".

And that leads us to a final little point, reconnecting with section 1.4 above where four epistemological styles, more general than those analyzed in 1.2 and 1.3, have been explored. In Table 5 they are related to the distinction between hard line and soft line approaches to contradictions of Figure 4:

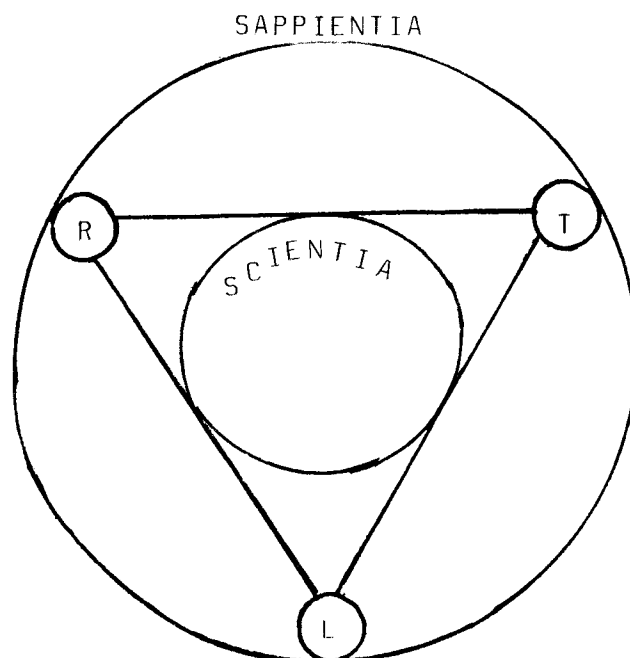
TABLE 5. Atomism/holism and deductivism/dialectics and the problem of contradiction/consistency

	hard line: deductivism	soft line: dialectics
atomism	seek and destroy	micro-dialectics
holism	structure-orientation	process-orientation

So, what do we find? At the level of atomism an opening for dialectics within the atom, be that the physical atom or the individual. Quantum mechanics and trendianism emerge; of course about at the same time. But at the level of holism contradictions become more dramatic as social processes: the case of marxism, in no way inconsistent with the dialectics of individuals, or atoms--or larger parts of matter as Prigogine would emphasize. A move from the upper left to the lower right--probably excessively so. The double dialectic of Table 5 is in itself valuable.

In conclusion, then, let us look at Ogden's triangle again. This time we present it with three corners, R for reality, T for thought, L for language, with an inner inscribed circle and an outer circle.

FIGURE 5. Ogden's triangle: Scientia & Sappientia



So, we have a narrow and a broad conceptualization of the relation between reality, thought, and language. In the narrow conceptualization reality is thought of as contradiction-free, thoughts are disciplined according to the Laws of thought. Language is consistent and since it is completely possible to be inconsistent in natural languages artificial languages are substituted for them, mathematics, logic and computer languages being examples of languages held to be consistent. The entire exercise is considered scientific, as scientia.

And then there is the outer circle of sappientia,¹⁰ of knowledge as insight in the broader sense. The basic notion of consistency is relaxed, but hardly totally. The world is seen

as more contradictory, thoughts are permitted, perhaps even encouraged to wander and wonder, languages of all kinds are fashioned accordingly, including less and less disciplined languages in the arts of space (painting, sculpture) and time (literature and music) and both time and space (theater, ballet).

Which reality is real?¹¹ Reality as captured by the inner circle or by the outer circle--or perhaps even further out, in the mist where reality, thought and language are very fleeting and floating indeed, perhaps to the point that they are no longer distinguishable--the three corners of Ogden's triangle folding back on themselves, so to speak? A glance at the presentation in 1.1 above of christian and buddhist epistemologies may perhaps to some readers point exactly to that: in buddhist epistemology there is no discipline at all, no rule of consistency, hence everything is permitted because there are no borderlines between reality, thought and language and no clear structure imputed to the three realms Ogden's triangle is constructed to keep apart.

In line with 1.4 above my own position on the questions stated would be eclectic: both-and. Why should we not permit ourselves to cast reality and our thoughts in a strait-jacket of consistency, and express our thoughts in languages constructed according to the rule of consistency? What we think of as "science" does exactly that, and the activity, like all other activities, should also be tested in terms of its consequences. The activity has given us brain surgery and atomic weapons.

Correspondingly, we could ask: why should we not permit ourselves to let everything go, to blur in the distinction between ourselves and a tree outside our window even to the point of asserting; like schizophrenic patients in a mental hospital located in some forest might say: "I am that tree, the tree is me". And we are in the sphere which has given us myths and fantasies, undisciplined art and speculation.

So again, which reality is more real? Who are we to judge? Majority vote? In a "competence group"? In what tradition has that competence group been socialized, within what civilization, what sub-civilization?

Can there be any other answer to that question than to open the field between the two circles in the figure and not only permit, even encourage people to oscillate between them, using one as a source of inspiration for the other?

The key corner to discuss in this connection seems to be language, as reality and thought are so inextricably interlinked in an an/sich/für mich dialectic. We are not in a position to impose or relax restrictions on reality. We may be more in a position to exercise thought control, but hardly able to control everybody all the time. So what we have succeeded in doing in the effort to create the edifice of science is the realm of language control, by imposing rules of consistency, creating "artificial" languages and then getting back at reality by the stipulation that reality is only understood "scientifically" when our thoughts about reality are expressed in contradiction-

free languages. The assumption has been that there is a direct link between rigor in language and rigor in thought, probably a terrible proposition, and rigor in both of these on the one hand and a rigorous presumably meaning valid, understanding of reality-- possibly an untenable proposition. One may then legitimately ask: what is more important, to exercise and demonstrate a particular skill in disciplined use of a consistent, rigorous language or to exercise, even demonstrate ability to say something meaningful about reality? Assuming that it is not obvious that one of these virtues implies the other?

Whatever one's stand on this issue it should become clear that contradiction-free mathematics is only a conspiracy, and more particularly an occidental one, to the extent that language is canonized as the most legitimate language for valid insights in reality, not as a language per se, with its undeniable esthetic charms and intellectual challenges!² The problem seems, however, mainly to arise insofar as mathematical theorems, not only terms are used. One could imagine a soft use of mathematics as a form of presentation, with matrices, graphs and what not. There may be some simple theorems,³ but without the full development of a mathematical apparatus because of the ever present possibility that this becomes a strait-jacket that may very well capture "reality", but only in a twisted and forced form, deprived of its richness in the form of inner contradictions, relating to a more real reality like the specimens exhibited in a natural museum for zoology, botany and geology relate to the animal, plant and mineral "kingdoms" in real nature. Nobody will deny that much can be learned from such museums, particu-

larly as long as one keeps the difference between museum reality and natural reality in mind.

Language, or form of presentation, then becomes a key issue. How do we discuss this issue? Which would be some of the key variables among forms of presentation, how do we select? To this we now turn, as the subject of the final chapter in this book, before the attempt to tie it all together in the epilogue. But one point can already be indicated from the epilogue: the need for some type of bridge building between scientia and sappientia which is relatively similar to the needs for some kind of bridge building between the famous two cultures by C. P. Snow.¹⁴ And one such bridge can be understood in terms of the hard and soft approaches to contradiction, being anchored in the inner circle in a total respect for consistency, reaching out to the outer circle as a mid-way position of accepting the dialectic between consistency and contradiction as everlasting, and then further out in the mist of total disrespect for consistency, even at the point of seeking contradiction for its own sake, not only coexisting passively with it. I am sure there are other bridges, but this was the bridge of concern for this little exercise.